INTRODUCTION

TO

MATHEMATICAL PROBABILITY

BY

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Professor of Mathematics, Stanford University

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NUMERICAL MATHEMATICAL ANALYSIS

BY

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of my son
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(1919-1927)

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(5)

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Table des matières

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DEDICATION

To all professors of the Université du Benin (Lomé, Togo), who have been arrested, whilst this book was in the press, for having fought for freedom of mind and expression.

This book is also dedicated to all those scientists who have become political victims because they had the courage to abandon their ivory tower of esoteric science and fight against political repression.

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(7)

STUDIES IN LINEAR AND NON-LINEAR PROGRAMMING

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Preface

The present introduction to functional analysis addresses students in mat matics and physics who have some basic knowledge in analysis and linear algel It grew out of the lectures which have been given by the authors several tim

The book is divided into four parts and an appendix. In Part I the necess notions and results on vector spaces, metric and topological spaces, as well on compact topological spaces, are provided.

In Part II we present the classical fundamentals of functional analysis. Af introducing Banach and Fréchet spaces we prove the Hahn–Banach theor and apply it and the bipolar theorem to study dual and bidual spaces as v as the closed range theorem. As consequences of Baire's theorem we prove open mapping theorem, the closed graph theorem and the principle of unifo boundedness. After introducing Hilbert spaces we deal with the spaces $L_p(X,$ and $C(X)'$ and we study Fourier transform and Sobolev spaces extensively.

Part III is devoted to the spectral theory of linear operators. Beginn with Riesz's theory of compact operators in Banach and Hilbert spaces discuss in detail Hilbert–Schmidt and trace class operators. The construct of spectral measures for normal operators in Hilbert spaces is prepared by chapter on Banach algebras where we also treat C^*-algebras and Gelfan theory. After proving the spectral representation for normal operators we dedu the corresponding result for (unbounded) self-adjoint operators from it usi the Cayley transform. Also, we present von Neumann's theory of self-adjo extensions of symmetric operators.

In Part IV we introduce locally convex spaces, their duality theory and characterize reflexive spaces. Further, we treat inductive and projective topo gies, Schwartz and (LF)-spaces as well as notions related to them and we pro the closed graph theorem of de Wilde. Then we concentrate on Fréchet and (DI spaces, where we also include recent results on the exactness of short sequences Fréchet spaces. Next a comprehensive presentation of the Köthe sequence spac illustrates many notions introduced so far and provides important examples a counter-examples. After a short introduction to nuclear spaces we systematica present power series spaces. Then we prove the (DN)-(Ω)-splitting theorem whi is closely related to power series spaces of infinite type and which is us to characterize the subspaces and the quotients of the space s of all rapic decreasing sequences.

In the appendix we give a short introduction to integration theory by mea of the Daniell integral so that spaces $L_p(X, \mu)$ and special integrals can be treat

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9

METHODS OF NONLINEAR ANALYSIS

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Departments of Mathematics,
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Los Angeles, California

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1970

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Richard Bellman

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I

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3

Exterior Ballistics

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3.1 Introduction

In studying the motion of a projectile through the atmosphere, the ballistician encounters a considerable variety of mathematical problems. It is the theme of the present chapter to describe as systematically as is consistent with the space available the various kinds of problems that arise and the kinds of mathematical equipment that one must have in order to solve them.

The motion of a projectile is generally thought of as being governed by a system of ordinary differential equations relating its position, velocity, and acceleration to the system of forces acting on it. Therefore one must expect to have to solve, in some fashion or other, fairly large systems of complicated differential equations. Before this part of the work is reached, however, an equally important and sometimes more difficult task must be accomplished, namely, that of setting up the proper equations of motion. This involves selecting suitable systems of coordinates in which to describe the motion of the projectile and developing a theory of what kinds of forces act on it. After a theory has been developed, it is usually possible to determine by experiment the actual magnitudes of the forces involved and thus to obtain definite equations of motion. For example, it is natural to theorize that the principal aerodynamic force on a projectile is in the direction opposite to the velocity and is a function of the velocity; experiment can determine that force for various projectiles and various velocities.

After the differential equations have been set up, it is usually found that, even with modern high-speed computing available, it is too great a problem to solve them for all the sets of physical situations and initial conditions that are likely to occur. For one thing, even the tabulating of and access to the results would be difficult if these results depended on

several dozen parameters. Therefore the ballistician constantly finds himself asking what approximations he can make, what forces he can ignore, what deviations from some standard ones are significant, and what are not. In ballistic problems up to now it has been possible to ignore the gravitational attraction of the sun, moon, and other heavenly bodies, but this may not continue to be the case in the future. Also he asks if something can safely be ignored or if it can temporarily be ignored and later corrected for in some simple manner. The most successful ballistician is likely to be the one who is the most skillful approximator.

In the rest of this chapter these points and others will be discussed in more detail.

3.2 Selection of Coordinate Systems

The selection of a coordinate system is not always quite as trivial a matter as it may seem. In the elementary mechanics of moving bodies, we usually place the origin of coordinates at some convenient point, such as the beginning of the trajectory, with the z axis vertical and the xy plane tangent to the earth's surface. We then proceed to write Newton's law, force equals mass times acceleration, forgetting, ignoring, or hoping that it does not matter that this coordinate system is moving as the earth rotates and travels around the sun. Now this motion is not uniform but involves acceleration, and so Newton's law is not valid for this system.

In order to see what kind of problem comes up in this connection, consider a very much simplified situation. Imagine a flat motionless earth and a railway train moving on it along the x axis. Suppose that the train is accelerating at a constant rate a and that at time $t = 0$ it is at the origin with zero velocity. At time $t = 0$ a ball is rolled forward on the frictionless train floor with initial velocity of magnitude v_0. This v_0 is with respect either to the train or to the ground, since at $t = 0$ the velocity of the train is zero.

Now two systems of coordinates suggest themselves: one on the ground and one moving with the train. If O' designates the point on the train floor from which the ball started, then O' has acceleration a, whence, by elementary calculus, $OO' = \frac{1}{2}at^2$. Thus the ground coordinate x and the train coordinate x' of the ball are related by the equation

$$x = x' + \frac{at^2}{2}$$

No forces act on the ball in the x direction, so that its equation of motion is

$$m\ddot{x} = 0 \tag{3.1}$$

Since $\ddot{x} = \ddot{x}' + a$, Eq. (3.1) becomes, in terms of x',

$$m\ddot{x}' = -ma \tag{3.2}$$

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Pietro Donatini · Patrizio Frosini

Natural pseudodistances between closed surfaces

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Abstract. Let us consider two closed surfaces $\mathcal{M}, \mathcal{N}$ of class C^1 and two functions $\varphi : \mathcal{M} \to \mathbb{R}$, $\psi : \mathcal{N} \to \mathbb{R}$ of class C^1, called measuring functions. The natural pseudodistance d between the pairs $(\mathcal{M}, \varphi)$, $(\mathcal{N}, \psi)$ is defined as the infimum of $\Theta(f) := \max_{P\in\mathcal{M}} |\varphi(P) - \psi(f(P))|$ as f varies in the set of all homeomorphisms from $\mathcal{M}$ onto $\mathcal{N}$. In this paper we prove that the natural pseudodistance equals either $|c_1 - c_2|$, $\frac{1}{2}|c_1 - c_2|$, or $\frac{1}{3}|c_1 - c_2|$, where c_1 and c_2 are two suitable critical values of the measuring functions. This shows that a previous relation between the natural pseudodistance and critical values obtained in general dimension can be improved in the case of closed surfaces. Our result is based on a theorem by Jost and Schoen concerning harmonic maps between surfaces.

Keywords. Natural pseudodistance, measuring function, harmonic map

Introduction

The *natural pseudodistance* is a new variational approach to the comparison of manifolds endowed with real-valued functions defined on them. In [2] we proved a result about the values that such a pseudodistance δ can take in general dimension. In this work we focus on the 2-dimensional case, showing that the previous result can be improved in the case of closed surfaces. Assuming that two homeomorphic closed manifolds $\mathcal{M}$ and $\mathcal{N}$ of class C^1 are given together with two functions $\varphi : \mathcal{M} \to \mathbb{R}$, $\psi : \mathcal{N} \to \mathbb{R}$ of class C^1 (called *measuring functions*), we consider the value

$$\delta((\mathcal{M}, \varphi), (\mathcal{N}, \psi)) := \inf_{f\in H(\mathcal{M},\mathcal{N})} \max_{P\in\mathcal{M}} |\varphi(P) - \psi(f(P))|,$$

where $H(\mathcal{M}, \mathcal{N})$ denotes the set of all homeomorphisms from $\mathcal{M}$ onto $\mathcal{N}$. The number $d = \delta((\mathcal{M}, \varphi), (\mathcal{N}, \psi))$ is called the *natural pseudodistance* between the pairs $(\mathcal{M}, \varphi)$ and $(\mathcal{N}, \psi)$ (called *size pairs*).

The closeness of d to zero means that there are homeomorphisms for which the difference between the values taken by the measuring functions at corresponding points is

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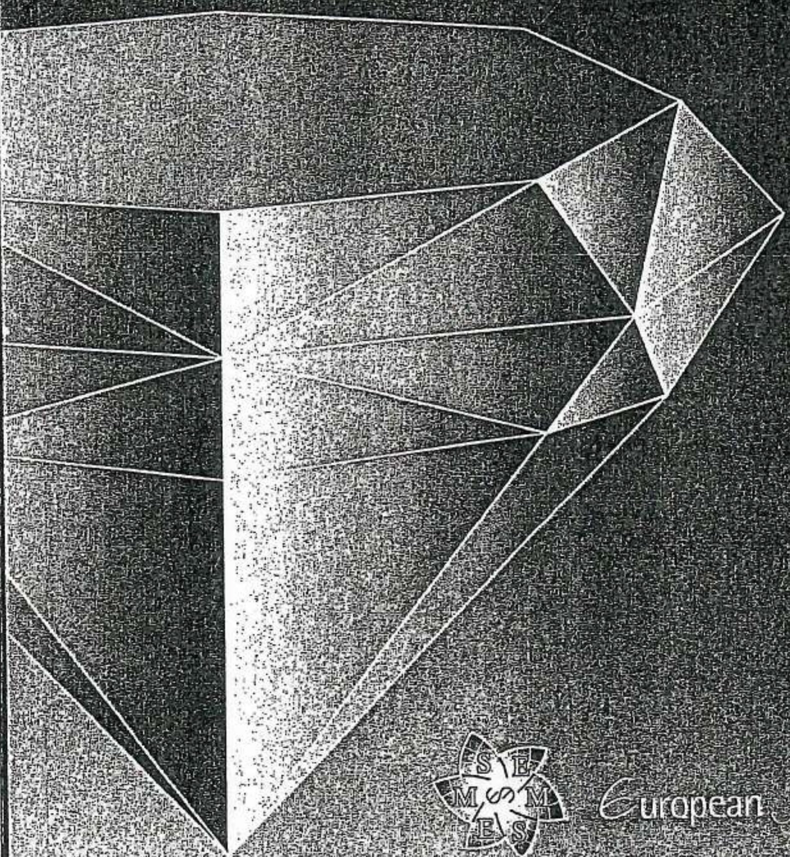

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continuation on page 251

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560

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PREFACE

The fourth Australian conference on combinatorial mathematics was held at the University of Adelaide from 27th to 29th August, 1975. The names of the fifty-two participants are listed overleaf.

The conference was fortunate enough to hear addresses given by distinguished combinatorialists from three countries: L.J. Cummings (Canada), D.A. Preece (England) and J.S. Wallis (Australia). This volume contains the texts of two of these invited addresses and of twenty-three contributed talks. Manuscripts of the five remaining contributed talks given at the conference are to be published elsewhere.

Many people helped with the organisation of the conference and with the publication of this volume and we are grateful to all of them. We particularly thank all those who chaired sessions and refereed papers. We thank the University of Adelaide for providing facilities, and in particular the staff of the University Union. Particular thanks are extended to Professor E.S. Barnes, the Deputy Vice-Chancellor.

Finally we thank Miss Anne Nicholls for typing this manuscript.

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SOME COMPUTATIONAL RESULTS ON THE SPECTRA OF GRAPHS

C. GODSIL AND B. MCKAY

The polynomial *of a graph is the characteristic polynomial of its 0-1 adjacency matrix. Two graphs are* cospectral *if their polynomials are the same.*

In this paper some of the results from a numerical study of the polynomials of graphs are presented. The study has encompassed 9 point graphs, 9 point bipartite graphs, 14 point trees and 13 point forests. Also given are several theoretical results which were prompted by the numerical data. These include two characterizations of those cospectral graphs which have cospectral complements, and a proof that, in the sense of Schwenk [20] "almost no" trees are characterized by their polynomials together with the polynomials of their complements. In addition, mention is made of those co-spectral graphs which have cospectral linegraphs, and those which are cospectral to their own complements.

1. INTRODUCTION

Graphs referred to in this paper have a finite, non-zero number of vertices and no loops or multiple edges. For such a graph G, $\overline{G}$ refers to the complement of G, and L(G) to the linegraph of G. For brevity, a graph on n vertices will be called an n-graph.

Suppose G is an n-graph. The *adjacency matrix* of G, also denoted G, is the $n \times n$ matrix whose (i,j)th entry is the number of edges from vertex i to vertex j. The *polynomial* of G, denoted $G(\lambda)$, is the characteristic polynomial of the adjacency matrix of G. An *eigenvalue* of G is a root of $G(\lambda)$. The eigenvalues of G, together with their multiplicities, constitute the *spectrum* of G. Two graphs which have the same polynomial, and hence the same spectrum are called *cospectral*.

Other graph theoretic concepts not defined here can be found in Harary [9] or in Behzad and Chartrand [2]. For any square matrix A, the trace of A is denoted tr A. J will always refer to a square matrix with each element one and I to an identity matrix.

The main purpose of this paper is to give the preliminary results of a computational study of the spectra of graphs. Previous studies of this kind have been made by Collatz and Singowitz [4] (5 point graphs and 8 point trees), King [13] (7 point graphs) and Mowshowitz [16] (10 point trees). In this study, the polynomials of 9 point graphs, 9 point bipartite graphs, 14 point trees and 13 point forests have been

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