

1

INTRODUCTION
TO
MATHEMATICAL PROBABILITY

BY
J. V. USPENSKY
Professor of Mathematics, Stanford University

FIRST EDITION
FIFTH IMPRESSION



McGRAW-HILL BOOK COMPANY, INC.
NEW YORK AND LONDON
1937

IX, 444 p.

(2)

NUMERICAL MATHEMATICAL ANALYSIS

BY

JAMES B. SCARBOROUGH, PH. D.

PROFESSOR EMERITUS OF MATHEMATICS

AT THE U. S. NAVAL ACADEMY

BALTIMORE: THE JOHNS HOPKINS PRESS

LONDON: OXFORD UNIVERSITY PRESS

1962

XXI, 546

COPYRIGHT 1930

BY

JAMES B. SCARBOROUGH

COPYRIGHT 1950, 1955, 1958, 1962

BY

THE JOHNS HOPKINS PRESS

SECOND EDITION, 1950

THIRD EDITION, 1955

FOURTH EDITION, 1958

FIFTH EDITION, 1962

*To the memory
of my son*

JAMES BLAINE SCARBOROUGH, JR.
(1919-1927)

Collection Enseignement des sciences

AN
NU
LIN

1. Henri Cartan
2. Jean Kuntzmann
3. Laurent Schwartz
4. Destouches et Cazin
5. Roger Godement
6. Jean-Marie Souriau
7. Gustave Choquet
8. Jean Dieudonné
9. Noël Gastinel

Théorie élémentaire des fonctions analytiques
Mathématiques de la physique et de la technique
Méthodes mathématiques pour les sciences physiques
Eléments de cinématique
Cours d'algèbre
Géométrie et relativité
L'enseignement de la géométrie
Algèbre linéaire et géométrie linéaire
Analyse numérique linéaire

Au
condi
plus
est é
suive
moye
cette
nouvi

Ce
ayant
nivea

L'a
de c
rithm
méth
par F
en A
ner é
tats é
L'étu
algori
mani
doit
certai
seigné
génér

L'o
collab
matiq
de G

Un voi
60 figu
relié i
Collect

Noël Gastinel

Professeur à la Faculté des sciences de Grenoble

3

Analyse numérique linéaire

Hermann, 115, boulevard Saint-Germain, Paris VI



XII, 362 p.

TABLE DES MATIÈRES

AVERTISSEMENT

CHAPITRE I: PROPRIÉTÉS ÉLÉMENTAIRES DES MATRICES

 I — Généralités

 II — Opérations sur les matrices

 1. Addition

 2. Multiplication par un nombre

 3. Produit de 2 matrices

 4. Produits matriciels particuliers

 5. Calcul numérique du produit matriciel

 6. Matrices déduites d'une matrice donnée

CHAPITRE II: NORMES DE VECTEURS ET DE MATRICES

 1. Définitions

 2. Exemples de normes de vecteurs

 3. Normes de matrices

 4. Procédure ALGOL

 5. Définitions de normes « géométriques »

 6. Normes « géométriques » de matrices

 7. Normes de matrices de $\mathcal{M}_{(n,n)}$ (matrices carrées)

 8. \mathbb{R}^n (ou \mathbb{C}^n) comme espaces de « Hilbert »

CHAPITRE III: INVERSION DES MATRICES — THÉORIE

 I — Indépendance linéaire de vecteurs dans un espace vectoriel

 1. Définitions

 2. Systèmes de générateurs

 3. Définition

 II — Théorème fondamental d'existence d'une solution d'un système linéaire homogène ayant plus d'équations que d'inconnues

 III — Dimension

 IV — Isomorphisme de \mathbb{R}^n (ou \mathbb{C}^n) à tout espace vectoriel sur \mathbb{R} (ou \mathbb{C}) de dimension finie n

 V — Inversibilité d'une application linéaire d'un \mathbb{R}^n dans un \mathbb{R}^m (ou \mathbb{C}^n dans \mathbb{C}^m)

 VI — Linéarité de l'application inverse d'une application linéaire inversible. Matrice inverse

 VII — Indicateur d'indépendance linéaire

© HERMANN, PARIS 1966

Tous droits de reproduction, même fragmentaire, sous quelque forme que ce soit, y compris photographie, photocopie, microfilm, bande magnétique, disque, ou autre, réservés pour tous pays. Toute reproduction, même partielle, non expressément autorisée, constitue une contrefaçon passible des peines prévues par la loi du 11 mars 1957 sur la protection des droits d'auteur.

DÉPARTEMENT DE MATHÉMATIQUES PURES
ET APPLIQUÉES

Directeur Général : André LICHNEROWICZ
Professeur au Collège de France

COLLECTION : TRAVAUX ET RECHERCHES MATHÉMATIQUES

Directeur : André LICHNEROWICZ

- I. — D. MASSIGNON. — *Mécanique statistique des fluides.*
- II. — A. LICHNEROWICZ. — *Théorie globale des connexions et des groupes d'holonomie.*
- III. — A. LICHNEROWICZ. — *Géométrie des groupes de transformation.*
- IV. — P. JAFFARD. — *Les systèmes d'idéaux.*
- V. — N. N. BOGOLIOUBOV et D. V. CHIRCHOV. — *Introduction à la théorie quantique des champs.*
- VI. — D. KASTLER. — *Introduction à l'électrodynamique quantique.*
- VII. — M. A. NAÏMARK. — *Les représentations linéaires du groupe de Lorentz.*

Sous presse :

- VIII. — Y. V. LINNIK. — *Méthode des moindres carrés.*

En préparation :

- IX. — P. DUBREIL et R. CROISOT. — *Demi-groupes.*
- X. — R. DEHEUVELS. — *Variétés et classes caractéristiques.*
- XI. — M. KERVAIRE. — *Théorie du type d'homotopie.*
- XII. — I. I. PIATETSKY-CHAIRO. — *Géométrie du domaine classique et des fonctions automorphes.*
- XIII. — L. S. PONTRIAGUINE, V. G. BOLTJANSKY, R. V. GAMKRELIDZÉ, E. MISHTCHENKO. — *Théorie mathématique des processus optimaux.*

COLLECTION UNIVERSITAIRE DE MATHÉMATIQUES

Directeur : Henri HIERCHE

- I. — M. ZAMANSKY. — *Introduction à l'algèbre de l'analyse moderne.*
- II. — C. BERGE. — *Théorie des graphes et ses applications.*
- III. — C. BERGE. — *Espaces topologiques et fonctions multivoques.*
- IV. — C. PISOT et M. ZAMANSKY. — *Mathématiques générales.*
- V. — L. FÉLIX. — *Exposé moderne des mathématiques élémentaires (2^e édition).*
- VI. — P. DUBREIL et DUBREIL-JACOTIN. — *Leçons d'algèbre moderne.*
- VII. — G. LEFORT. — *Algèbre et analyse. Exercices.*
- VIII. — I. M. GUELFAND et G. E. CHILOV. — *Les distributions.*
- IX. — H. CABANNES. — *Cours de mécanique générale.*
- X. — D. PHAM, avec la collaboration de M. GHINEA. — *Techniques du calcul matriciel.*

Sous pressé :

- XI. — E. B. DYNKIN. — *Théorie des processus markoviens.*
- XII. — A. O. GUELFOND. — *Calcul des différences finies.*

En préparation :

- XIII. — P. S. NOVIKOV. — *Introduction à la logique mathématique.*
- XIV. — B. V. GNIEDENKO. — *Théorie des probabilités.*
- XV. — F. R. GANTMACHER. — *Théorie des matrices.*
- XVI. — R. FORTET. — *Processus stochastiques.*
- XVII. — I. M. GUELFAND et G. E. CHILOV. — *Distributions et espaces fondamentaux.*
- XVIII. — I. M. GUELFAND et G. E. CHILOV. — *Distributions et théorie des équations différentielles.*
- XIX. — I. M. GUELFAND et N. Y. VILENKIN. — *Distributions et applications de l'analyse harmonique.*

COLLECTION UNIVERSITAIRE DE MATHÉMATIQUES

Daniel PHAM

Professeur à la Faculté des Sciences de Caen

TECHNIQUES
DU
CALCUL MATRICIEL

AVEC LA COLLABORATION DE

Monique GHINEA

Docteur ès Sciences
Chargée d'Enseignement au C. S. U. du Mans

PRÉFACE DE

A. LICHNEROWICZ

Professeur au Collège de France

DUNOD
PARIS
1962

Institut de Mathématique
BIBLIOTHÈQUE
15, Av. des Tilleuls, LIÈGE (Belgique)

XIII, 2738

4
AIRE
JES
ES
CIEL
de Caen
ON DI
du Man
CZ
rance
bot
ous diff
par di
contien
plus con
re l'espr
ticiens
pour
eurs. D
complè
s théori
tions pr
employ
rendre
ceux q
me nun
intéress
ématique
é tous
is ayant
ectroniq
recherch
deul nun
at ou à
au cours
s du cal
R. PAF

CHEZ LE MÊME ÉDITEUR

- MATHÉMATIQUES POUR LA LICENCE, second cycle des Universités et des écoles d'ingénieurs. Variable complexe, calcul différentiel et tensoriel, espaces normés et calcul intégral, analyse de Fourier, par J.P. FERRIER. 1984, 232 pages.
- INTÉGRATION ET PROBABILITÉS, ANALYSE DE FOURIER ET ANALYSE SPECTRALE, par P. MAL-
LIAVIN. *Maîtrise de Mathématiques Pures*. 1982, 200 pages.
- INTÉGRATION ET PROBABILITÉS, ANALYSE DE FOURIER ET ANALYSE SPECTRALE. EXERCICES, par
G. LETAC. *Maîtrise de Mathématiques Pures*. 1982, 152 pages.
- RÉSOLUTION NUMÉRIQUE DES ÉQUATIONS AUX DÉRIVÉES PARTIELLES, de la physique, de
la mécanique et des sciences de l'ingénieur. Différences finies, Eléments finis, par
D. EUVRARD. *Enseignement de la Physique*. 2^e édition 1990, 342 pages.
- L'OUTIL MATHÉMATIQUE. Distributions. Convolution. Transformations de Fourier et de
Laplace. Fonction d'une variable complexe, Fonction Eulérienne, par R. PETIT. *Ensei-
gnement de la physique*. 3^e édition, 1990, 272 pages.
- ANALYSE MATHÉMATIQUE ET CALCUL NUMÉRIQUE POUR LES SCIENCES ET LES TECHNIQUES, sous
la direction de R. DAUTRAY et J.L. LIONS. *Commissariat à l'Énergie Atomique, série
scientifique*.
Volume 1. – Modèles physiques. 1987, 296 pages.
Volume 2. – L'opérateur de Laplace. 1987, 566 pages.
Volume 3. – Transformations, Sobolev, opérateurs. 1987, 320 pages.
Volume 4. – Méthodes variationnelles. 1988, 354 pages.
Volume 5. – Spectres des opérateurs. 1988, 616 pages.
Volume 6. – Méthodes intégrales et numériques. 1988, 550 pages.
Volume 7. – Évolution : Fourier, Laplace. 1988, 412 pages.
Volume 8. – Évolution : semi-groupe, variationnel. 1988, 578 pages.
Volume 9. – Évolution : numérique, transport. 1988, 498 pages.
- ANALYSE COMPLEXE, par P. DOLBEAULT. *Maîtrise de Mathématiques Pures*. 1990, 252 pages.
- ANALYSE RÉELLE ET COMPLEXE, par W. RUDIN. Traduit de l'anglais par N. DHOMBRES et
F. HOFFMAN. 1987, 408 pages.
- TRAITEMENT NUMÉRIQUE DU SIGNAL. Théorie et pratique, par M. BELLANGER. *Collection
Technique et Scientifique des télécommunications* 1990, 464 pages.
- THÉORIE DU SIGNAL. Modélisation statistique, automatique et traitement, par D. DE BRUCQ
et G. FOLLIO. 1988, 488 pages.
- CODAGE ET TRAITEMENT DU SIGNAL. L'exemple des systèmes vidéonumériques, par
J.G. WADE. Traduit de l'anglais par J.L. MAUREL et I. BRESIT. 1990, 256 pages.
- L'ANALYSE HARMONIQUE. Son développement historique, par J.P. PIER. 1990, 344 pages.

5

Analyse de Fourier et applicatic

Filtrage, calcul numérique, ondelettes

Claude GASQUET

Université Joseph Fourier (Grenoble I)

Patrick WITOMSKI

Université Joseph Fourier (Grenoble I)

XI, 354p.

MASSON
Paris Milan Barcelone Mexico
1990

Table des matières

Les cartes de tarot qui figurent dans cet ouvrage ont été reproduites avec l'aimable autorisation de la Société France Cartes – Maître Cartier à Saint-Max (54130)

Tous droits de traduction, d'adaptation et de reproduction par tous procédés réservés pour tous pays.

Toute reproduction ou représentation intégrale ou partielle, par quelque procédé que ce soit, des pages publiées dans le présent ouvrage, faite sans l'autorisation de l'éditeur est illicite et constitue une contrefaçon. Seules sont autorisées, d'une part, les reproductions strictement réservées à l'usage privé du copiste et non destinées à une utilisation collective, et d'autre part, les courtes citations justifiées par le caractère scientifique ou d'information de l'œuvre dans laquelle elles sont incorporées (loi du 11 mars 1957 art. 40 et 41 et Code pénal art. 425).

Des photocopies payantes peuvent être réalisées avec l'accord de l'éditeur. S'adresser au : Centre français du copyright, 6 bis, rue Gabriel-Laumain, 75010 Paris. Tél. : 48.24.98.30.

© Masson, Paris, 1990

ISBN : 2-225-82018-X

MASSON
MASSON S.p.A.
MASSON S.A.
MASSON EDITORES

120, bd Saint-Germain, 75280 Paris Cedex 06
Via Statuto 2/4, 20121 Milano
Balmes 151, 08008 Barcelona
Dakota 383, Colonia Napoles, 03810 Mexico D.F.

Avant-propos	
Images	
Chapitre I. – Signaux et systèmes	
LEÇON n° 1. – Où il est question de signaux et de systèmes	
1.1. Généralités	
1.2. Quelques signaux élémentaires	
1.3. Exemples de systèmes	
LEÇON n° 2. – Où les filtres font des transferts	
2.1. Propriétés algébriques des systèmes	
2.2. La continuité d'un système	
2.3. Le filtre et sa fonction de transfert	
2.4. Un filtre analogique standard : la cellule RC	
2.5. Cas d'un filtre discret du premier ordre	
Chapitre II. – Signaux périodiques	
LEÇON n° 3. – Les signaux trigonométriques	
3.1. Polynômes trigonométriques	
3.2. Représentation en sinus et cosinus	
3.3. Propriété d'orthogonalité	
LEÇON n° 4. – Etude d'un signal périodique quelconque. Séries de Fourier	
4.1. Le cadre de l'espace $L^2_P(0, a)$	
4.2. L'idée d'approximation	
4.3. Convergence de l'approximation	
4.4. Coefficients de Fourier d'une fonction réelle, paire, impaire	
4.5. Formulaire	
LEÇON n° 5. – Le problème de la représentation ponctuelle d'une fonction par sa série de Fourier	
5.1. Le théorème de Riemann-Lebesgue	
5.2. Convergence ponctuelle ?	
5.3. Convergence uniforme de la série de Fourier	

6

STUDIES IN MATHEMATICS AND ITS APPLICATIONS

VOLUME 3

Editors:

J. L. LIONS, *Paris*
G. PAPANICOLAOU, *New York*
R. T. ROCKAFELLAR, *Seattle*



DIFFERENTIAL FORMS IN MATHEMATICAL PHYSICS

C. VON WESTENHOLZ
Université du Benin (Togo)
and
University of Zambia

Institut de Mathématique
BIBLIOTHÈQUE
15, avenue des Tiléuls — B-4000 LIÈGE



1978

XV, 487p.

NORTH-HOLLAND PUBLISHING COMPANY
AMSTERDAM · NEW YORK · OXFORD

© North-Holland Publishing Company 1978

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording or otherwise, without the prior permission of the copyright owner.

Published by:
North-Holland Publishing Company
Amsterdam · New York · Oxford

Sole distributors for the U.S.A. and Canada:
Elsevier North-Holland, Inc.
52 Vanderbilt Avenue
New York, NY 10017

Library of Congress Cataloging in Publication Data

Westenholz, Cornelius von.
Differential forms in mathematical physics.
(Studies in mathematics and its applications; v. 3)

Bibliography: p.
Includes index.
1. Differential forms. 2. Mathematical for
physics. I. Title.
QC20.7.D52W47 530.1'5'636 76-48255
ISBN 0-7204-0537-8

PRINTED IN THE NETHERLANDS

DEDICATION

To all professors of the Université du Bénin (Lomé, Togo),
who have been arrested, whilst this book was in
the press, for having fought for freedom of mind
and expression.

This book is also dedicated to all those scientists
who have become political victims because they had
the courage to abandon their ivory tower of esoteric
science and fight against political repression.



STUDIES IN
LINEAR AND
NON-LINEAR
PROGRAMMING

by

KENNETH J. ARROW
LEONID HURWICZ
HIROFUMI UZAWA

with contributions by

HOLLIS C. CHENEY
SELMER M. JOHNSON
SAMUEL KARLIN
THOMAS MARSHALL
ROBERT M. SOLOW

STANFORD MATHEMATICAL STUDIES
IN THE SOCIAL SCIENCES, II

Editors

KENNETH J. ARROW
SAMUEL KARLIN
PATRICK SUPPES

STANFORD UNIVERSITY PRESS
STANFORD, CALIFORNIA
1958

Université de Liège
Faculté des Sciences appliquées
Recherche opérationnelle appliquée

229 p.

OXFORD GRADUATE TEXTS IN MATHEMATICS

1. Keith Hannabuss: *An introduction to quantum theory*
2. Reinhold Meise and Dietmar Vogt: *Introduction to functional analysis*
3. James G. Oxley: *Matroid theory*

S 5574

8

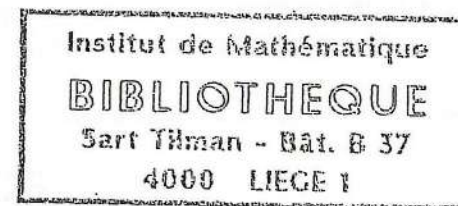
Introduction to Functional Analysis

Reinhold Meise
Mathematical Institute
Heinrich Heine University, Düsseldorf

and

Dietmar Vogt
Mathematics Faculty
University of Wuppertal

Translated by
M. S. Ramanujan



X, 437 p.

CLARENDON PRESS • OXFORD

Great Clarendon Street, Oxford OX2 6DP
Oxford New York
Athens Auckland Bangkok Bogota Bombay Buenos Aires
Calcutta Cape Town Dar es Salaam Delhi Florence Hong Kong
Istanbul Karachi Kuala Lumpur Madras Madrid Melbourne
Mexico City Nairobi Paris Singapore Taipei Tokyo Toronto Warsaw
and associated companies in
Berlin Ibadan

Oxford is a trade mark of Oxford University Press

Published in the United States by
Oxford University Press Inc., New York

Originally published in the German language by
Friedr. Vieweg & Sohn Verlagsgesellschaft mbH,
D-65183 Wiesbaden, Germany, under the title
'Einführung in die Funktionalanalysis. 1. Auflage (1st Edition)
© Friedr. Vieweg & Sohn Verlagsgesellschaft mbH,
Braunschweig/Wiesbaden, 1992
This edition
© R. Meise and D. Vogt, 1997

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, without the prior permission in writing of Oxford University Press. Within the UK, exceptions are allowed in respect of any fair dealing for the purpose of research or private study, or criticism or review, as permitted under the Copyright, Designs and Patents Act, 1988, or in the case of reprographic reproduction in accordance with the terms of licences issued by the Copyright Licensing Agency. Enquiries concerning reproduction outside those terms and in other countries should be sent to the Rights Department, Oxford University Press, at the address above.

This book is sold subject to the condition that it shall not, by way of trade or otherwise, be lent, re-sold, hired out, or otherwise circulated without the publisher's prior consent in any form of binding or cover other than that in which it is published and without a similar condition including this condition being imposed on the subsequent purchaser.

A catalogue record for this book is available from the British Library

Library of Congress Cataloging in Publication Data
(Data available)

ISBN 0 19 851485 9

Typeset by the translator using *Latex*

Printed in Great Britain by
Biddles Ltd, Guildford and King's Lynn

Preface

The present introduction to functional analysis addresses students in mathematics and physics who have some basic knowledge in analysis and linear algebra. It grew out of the lectures which have been given by the authors several times.

The book is divided into four parts and an appendix. In Part I the necessary notions and results on vector spaces, metric and topological spaces, as well as on compact topological spaces, are provided.

In Part II we present the classical fundamentals of functional analysis. After introducing Banach and Fréchet spaces we prove the Hahn–Banach theorem and apply it and the bipolar theorem to study dual and bidual spaces as well as the closed range theorem. As consequences of Baire's theorem we prove the open mapping theorem, the closed graph theorem and the principle of uniform boundedness. After introducing Hilbert spaces we deal with the spaces $L_p(X)$ and $C(X)'$ and we study Fourier transform and Sobolev spaces extensively.

Part III is devoted to the spectral theory of linear operators. Beginning with Riesz's theory of compact operators in Banach and Hilbert spaces we discuss in detail Hilbert–Schmidt and trace class operators. The construction of spectral measures for normal operators in Hilbert spaces is prepared by a chapter on Banach algebras where we also treat C^* -algebras and Gelfand theory. After proving the spectral representation for normal operators we deduce the corresponding result for (unbounded) self-adjoint operators from it using the Cayley transform. Also, we present von Neumann's theory of self-adjoint extensions of symmetric operators.

In Part IV we introduce locally convex spaces, their duality theory and characterize reflexive spaces. Further, we treat inductive and projective topologies, Schwartz and (LF)-spaces as well as notions related to them and we prove the closed graph theorem of de Wilde. Then we concentrate on Fréchet and (DF)-spaces, where we also include recent results on the exactness of short sequences of Fréchet spaces. Next a comprehensive presentation of the Köthe sequence space illustrates many notions introduced so far and provides important examples and counter-examples. After a short introduction to nuclear spaces we systematically present power series spaces. Then we prove the (DN)-(Ω)-splitting theorem which is closely related to power series spaces of infinite type and which is used to characterize the subspaces and the quotients of the space s of all rapidly decreasing sequences.

In the appendix we give a short introduction to integration theory by means of the Daniell integral so that spaces $L_p(X, \mu)$ and special integrals can be treated

9

METHODS OF NONLINEAR ANALYSIS

Richard Bellman

Departments of Mathematics,
Electrical Engineering, and Medicine
University of Southern California
Los Angeles, California

VOLUME I

Université de Liège
BST - Sciences Appliquées et Mathématiques
1. Chemin des Chevreuils, Bât B52/4
B-4000 LIEGE

This is Volume 61 in
MATHEMATICS IN SCIENCE AND ENGINEERING
A series of monographs and textbooks
Edited by RICHARD BELLMAN, *University of Southern California*

A complete list of the books in this series appears at the end of this volume.



ACADEMIC PRESS New York and London

METHODS OF NONLINEAR ANALYSIS

Richard Bellman

Departments of Mathematics,
Electrical Engineering, and Medicine
University of Southern California
Los Angeles, California

VOLUME II

Université de Liège
BST - Sciences Appliquées et Mathématiques
1, Chemin des Chevreuils; Bât B52/4
B-4000 LIEGE

This is Volume 61-II in
MATHEMATICS IN SCIENCE AND ENGINEERING
A series of monographs and textbooks
Edited by RICHARD BELLMAN, *University of Southern California*

The complete listing of books in this series is available from the Publisher
upon request.



ACADEMIC PRESS New York and London

Vol
This
stud
tech
and
prob
ous
whic
over
and
trem
ter.
a ca
and
icall
of t
The
appl
tion
equi
cist
s
ead
only
russ
ron
ill
unc
pur
thy
vill
on

10

METHODS OF NONLINEAR ANALYSIS

Richard Bellman

Departments of Mathematics,
Electrical Engineering, and Medicine
University of Southern California
Los Angeles, California

VOLUME II

Université de Liège
BST - Sciences Appliquées et Mathématiques
1, Chemin des Chevreuils; Bât B52/4
B-4000 LIEGE

This is Volume 61-II in
MATHEMATICS IN SCIENCE AND ENGINEERING
A series of monographs and textbooks
Edited by RICHARD BELLMAN, *University of Southern California*

The complete listing of books in this series is available from the Publisher upon request.



ACADEMIC PRESS New York and London

XVII, 261p

ISBN 0-12-084902-X

715
①

INTERNATIONAL SERIES OF MONOGRAPHS IN

PURE AND APPLIED MATHEMATICS

GENERAL EDITORS: I. N. SNEDDON, M. STARK AND S. ULAM

VOLUME 57

A COURSE OF
HIGHER MATHEMATICS

I

ELEMENTARY CALCULUS

XIII, 546P.

r
I
;
n
n
n
o
n
ty
e
s
F
an
o
of
ip
is
un
an
f
d
ys
as
nt
on
oi
he
T
ui
inc
g
th
ol

A COURSE OF Higher Mathematics

VOLUME I

V. I. SMIRNOV

Translated by
D. E. BROWN

*Translation edited and
additions made by*
I. N. SNEDDON
*Simson Professor in Mathematics
University of Glasgow*

PERGAMON PRESS
OXFORD · LONDON · EDINBURGH · NEW YORK
PARIS · FRANKFURT
1964

S
r
M
m
ov
m
vo
in
lit
ce
is,
s.
re
w
C
m
en
co
s i
se
ny
pr
es
c
n i
ite
or
jer
ts
JY,
er
je
c fl

PERGAMON PRESS LTD.
Headington Hill Hall, Oxford
4 & 5 Fitzroy Square, London W.1

PERGAMON PRESS (SCOTLAND) LTD.
2 & 3 Teviot Place, Edinburgh 1

PERGAMON PRESS INC.
122 East 55th Street, New York 22, N. Y.

GAUTHIER-VILLARS ED.
55 Quai des Grands-Augustins, Paris 6

PERGAMON PRESS G.m.b.H.
Kaiserstrasse 75, Frankfurt am Main

U.S. A. edition distributed by
ADDISON-WESLEY PUBLISHING COMPANY INC.
Reading, Massachusetts · Palo Alto · London

Copyright © 1964
PERGAMON PRESS LTD.

Library of Congress Catalog Card Number 63-10134

This translation has been made from the Sixteenth (revised) Russian Edition of
V. I. Smirnov's book *Курс высшей математики (Kurs vysshei matematiki)*,
published in 1957 by Fizmatgiz, Moscow

MADE IN GREAT BRITAIN

CONTENTS

INTRODUCTION	xi
PREFACES TO EIGHTH AND SIXTEENTH RUSSIAN EDITIONS	xiii

CHAPTER I

FUNCTIONAL RELATIONSHIPS AND THE THEORY OF LIMITS

§ 1. Variables	1
1. Magnitude and its measurement. 2. Number. 3. Constants and variables. 4. Interval. 5. The concept of function. 6. The analytic method of representing functional relationships. 7. Implicit functions. 8. The tabular method. 9. The graphical method of representing numbers. 10. Coordinates. 11. Graphs. The equation of a curve. 12. Linear functions. 13. Increment. The basic property of a linear function. 14. Graph of uniform motion. 15. Empirical formulae. 16. Parabola of the second degree. 17. Parabola of the third degree. 18. The law of inverse proportionality. 19. Power functions. 20. Inverse functions. 21. Many-valued functions. 22. Exponential and logarithmic functions. 23. Trigonometric functions. 24. Inverse trigonometric, or circular, functions.	
§ 2. The theory of limits. Continuous functions	43
25. Ordered variables. 26. Infinitesimals. 27. The limit of a variable. 28. Basic theorems. 29. Infinitely large magnitudes. 30. Monotonic variables. 31. Cauchy's test for the existence of a limit. 32. Simultaneous variation of two variables, connected by a functional relationship. 33. Example. 34. Continuity of functions. 35. The properties of continuous functions. 36. Comparison of infinitesimals and of infinitely large magnitudes. 37. Examples. 38. The number e. 39. Unproved hypotheses. 40. Real numbers. 41. The operations on real numbers. 42. The strict bounds of numerical sets. Tests for the existence of a limit. 43. Properties of continuous functions. 44. Continuity of elementary functions.	

EXERCISES	94
-----------	----

CHAPTER II

DIFFERENTIATION: THEORY AND APPLICATIONS

§ 3. Derivatives and differentials of the first order	101
45. The concept of derivative. 46. Geometrical significance of the derivative. 47. Derivatives of some simple functions. 48. Derivatives	

UNIVERSITY OF CALIFORNIA
ENGINEERING EXTENSION SERIES

Beckenbach · Modern Mathematics for the Engineer
Ridenour · Modern Physics for the Engineer
Robertson · Modern Chemistry for the Engineer and Scientist

In preparation:

Huberty and Flock · Natural Resources

19

MODERN MATHEMATICS FOR THE ENGINEER

ROYAL WELLER
SOLOMON LEFSCHETZ
RICHARD BELLMAN
JOHN W. GREEN
MAGNUS R. HESTENES
RICHARD COURANT
MENAHEM M. SCHIFFER
IVAN S. SOKOLNIKOFF
NORBERT WIENER
H. FREDERIC BOHNENBLUST
GILBERT W. KING
GEORGE W. BROWN
LOUIS A. PIPES
JOHN L. BARNES
EDWIN F. BECKENBACH
CHARLES B. MORREY, JR.
GEORGE E. FORSYTHE
CHARLES B. TOMPKINS
DERRICK H. LEHMER

Edited by
EDWIN F. BECKENBACH
Professor of Mathematics
University of California
Los Angeles

1956

McGraw-Hill Book Company, Inc.
NEW YORK TORONTO LONDON

XX, 514P

3

Exterior Ballistics

JOHN W. GREEN

PROFESSOR OF MATHEMATICS
UNIVERSITY OF CALIFORNIA, LOS ANGELES

3.1 Introduction

In studying the motion of a projectile through the atmosphere, the ballisticians encounters a considerable variety of mathematical problems. It is the theme of the present chapter to describe as systematically as is consistent with the space available the various kinds of problems that arise and the kinds of mathematical equipment that one must have in order to solve them.

The motion of a projectile is generally thought of as being governed by a system of ordinary differential equations relating its position, velocity, and acceleration to the system of forces acting on it. Therefore one must expect to have to solve, in some fashion or other, fairly large systems of complicated differential equations. Before this part of the work is reached, however, an equally important and sometimes more difficult task must be accomplished, namely, that of setting up the proper equations of motion. This involves selecting suitable systems of coordinates in which to describe the motion of the projectile and developing a theory of what kinds of forces act on it. After a theory has been developed, it is usually possible to determine by experiment the actual magnitudes of the forces involved and thus to obtain definite equations of motion. For example, it is natural to theorize that the principal aerodynamic force on a projectile is in the direction opposite to the velocity and is a function of the velocity; experiment can determine that force for various projectiles and various velocities.

After the differential equations have been set up, it is usually found that, even with modern high-speed computing available, it is too great a problem to solve them for all the sets of physical situations and initial conditions that are likely to occur. For one thing, even the tabulating of and access to the results would be difficult if these results depended on

13

several dozen parameters. Therefore the ballisticians constantly finds himself asking what approximations he can make, what forces he can ignore, what deviations from some standard ones are significant, and what are not. In ballistic problems up to now it has been possible to ignore the gravitational attraction of the sun, moon, and other heavenly bodies, but this may not continue to be the case in the future. Also he asks if something can safely be ignored or if it can temporarily be ignored and later corrected for in some simple manner. The most successful ballisticians is likely to be the one who is the most skillful approximator.

In the rest of this chapter these points and others will be discussed in more detail.

3.2 Selection of Coordinate Systems

The selection of a coordinate system is not always quite as trivial a matter as it may seem. In the elementary mechanics of moving bodies, we usually place the origin of coordinates at some convenient point, such as the beginning of the trajectory, with the z axis vertical and the xy plane tangent to the earth's surface. We then proceed to write Newton's law, force equals mass times acceleration, forgetting, ignoring, or hoping that it does not matter that this coordinate system is moving as the earth rotates and travels around the sun. Now this motion is not uniform but involves acceleration, and so Newton's law is not valid for this system.

In order to see what kind of problem comes up in this connection, consider a very much simplified situation. Imagine a flat motionless earth and a railway train moving on it along the x axis. Suppose that the train is accelerating at a constant rate a and that at time $t = 0$ it is at the origin with zero velocity. At time $t = 0$ a ball is rolled forward on the frictionless train floor with initial velocity of magnitude v_0 . This v_0 is with respect either to the train or to the ground, since at $t = 0$ the velocity of the train is zero.

Now two systems of coordinates suggest themselves: one on the ground and one moving with the train. If O' designates the point on the train floor from which the ball started, then O' has acceleration a , whence, by elementary calculus, $OO' = \frac{1}{2}at^2$. Thus the ground coordinate x and the train coordinate x' of the ball are related by the equation

$$x = x' + \frac{at^2}{2}$$

No forces act on the ball in the x direction, so that its equation of motion is

$$m\ddot{x} = 0 \tag{3.1}$$

Since $\dot{x} = \dot{x}' + a$, Eq. (3.1) becomes, in terms of x' ,

$$m\ddot{x}' = -ma \tag{3.2}$$

p. 36 -- 58

UNIVERSITY OF CALIFORNIA
ENGINEERING EXTENSION SERIES

Beckenbach · Modern Mathematics for the Engineer
Ridenour · Modern Physics for the Engineer
Robertson · Modern Chemistry for the Engineer and Scientist

In preparation:

Huberty and Flock · Natural Resources

MODERN MATHEMATICS FOR THE ENGINEER

ROYAL WELLER
SOLOMON LEFSCHETZ
RICHARD BELLMAN
JOHN W. GREEN
MAGNUS R. HESTENES
RICHARD COURANT
MENAHEM M. SCHIFFER
IVAN S. SOKOLNIKOFF
NORBERT WIENER
H. FREDERIC BOHNENBLUST
GILBERT W. KING
GEORGE W. BROWN
LOUIS A. PIPES
JOHN L. BARNES
EDWIN F. BECKENBACH
CHARLES B. MORREY, JR.
GEORGE E. FORSYTHE
CHARLES B. TOMPKINS
DERRICK H. LEHMER

Edited by
EDWIN F. BECKENBACH

Professor of Mathematics
University of California
Los Angeles

1956

McGraw-Hill Book Company, Inc.
NEW YORK TORONTO LONDON



Pietro Donatini · Patrizio Frosini

Natural pseudodistances between closed surfaces

Received October 13, 2005 and in revised form October 28, 2005

Abstract. Let us consider two closed surfaces \mathcal{M}, \mathcal{N} of class C^1 and two functions $\varphi : \mathcal{M} \rightarrow \mathbb{R}$, $\psi : \mathcal{N} \rightarrow \mathbb{R}$ of class C^1 , called measuring functions. The natural pseudodistance d between the pairs (\mathcal{M}, φ) , (\mathcal{N}, ψ) is defined as the infimum of $\Theta(f) := \max_{P \in \mathcal{M}} |\varphi(P) - \psi(f(P))|$ as f varies in the set of all homeomorphisms from \mathcal{M} onto \mathcal{N} . In this paper we prove that the natural pseudodistance equals either $|c_1 - c_2|$, $\frac{1}{2}|c_1 - c_2|$, or $\frac{1}{3}|c_1 - c_2|$, where c_1 and c_2 are two suitable critical values of the measuring functions. This shows that a previous relation between the natural pseudodistance and critical values obtained in general dimension can be improved in the case of closed surfaces. Our result is based on a theorem by Jost and Schoen concerning harmonic maps between surfaces.

Keywords. Natural pseudodistance, measuring function, harmonic map

Introduction

The *natural pseudodistance* is a new variational approach to the comparison of manifolds endowed with real-valued functions defined on them. In [2] we proved a result about the values that such a pseudodistance δ can take in general dimension. In this work we focus on the 2-dimensional case, showing that the previous result can be improved in the case of closed surfaces. Assuming that two homeomorphic closed manifolds \mathcal{M} and \mathcal{N} of class C^1 are given together with two functions $\varphi : \mathcal{M} \rightarrow \mathbb{R}$, $\psi : \mathcal{N} \rightarrow \mathbb{R}$ of class C^1 (called *measuring functions*), we consider the value

$$\delta((\mathcal{M}, \varphi), (\mathcal{N}, \psi)) := \inf_{f \in H(\mathcal{M}, \mathcal{N})} \max_{P \in \mathcal{M}} |\varphi(P) - \psi(f(P))|,$$

where $H(\mathcal{M}, \mathcal{N})$ denotes the set of all homeomorphisms from \mathcal{M} onto \mathcal{N} . The number $d = \delta((\mathcal{M}, \varphi), (\mathcal{N}, \psi))$ is called the *natural pseudodistance* between the pairs (\mathcal{M}, φ) and (\mathcal{N}, ψ) (called *size pairs*).

The closeness of d to zero means that there are homeomorphisms for which the difference between the values taken by the measuring functions at corresponding points is

P. Donatini: Dipartimento di Matematica, Università di Bologna, Piazza Porta San Donato, 5, I-40126 Bologna, Italy; e-mail: donatini@dm.unibo.it

P. Frosini: Advanced Research Centre for Electronic Systems “Ercole De Castro”, Università di Bologna, via Toffano, 2/2, I-40135 Bologna, Italy; e-mail: frosini@dm.unibo.it

100

Gidas, B., Ni, W.-M., Nirenberg, L.: Symmetry and related properties via the maximum principle. *Comm. Math. Phys.* **68**, 209–243 (1979) Zbl 0425.35020 MR 0544879

Li, C.: Monotonicity and symmetry of solutions of fully nonlinear elliptic equations. *Comm. Partial Differential Equations* **16**, 491–526 and 585–615 (1991) Zbl 0741.35014 MR 1113099

Pucci, C.: Operatori ellittici estremanti. *Ann. Mat. Pura Appl.* **72**, 141–170 (1966) Zbl 0154.12402 MR 0208150

Quaas, A.: Existence of positive solutions to a “semilinear” equation involving Pucci’s operator in a convex domain. *Differential Integral Equations* **17**, 481–494 (2004) MR 2054930

Quaas, A., Sirakov, B.: Existence results for nonproper elliptic equations involving the Pucci operator. *Comm. Partial Differential Equations* **31**, 987–1003 (2006) Zbl pre05062563 MR 2254600

Quaas, A., Sirakov, B.: On the principal eigenvalue and the Dirichlet problem for fully nonlinear operators. *C. R. Math. Acad. Sci. Paris* **342**, 115–118 (2006) Zbl pre02248352 MR 2193657

Quaas, A., Sirakov, B.: Existence and properties of a principal eigenvalue of fully nonlinear elliptic operators. Preprint

Reichel, W.: Radial symmetry for elliptic boundary-value problems on exterior domains. *Arch. Ration. Mech. Anal.* **137**, 381–394 (1997) Zbl 0891.35006 MR 1463801

Serrin, J.: A symmetry theorem in potential theory. *Arch. Ration. Mech. Anal.* **43**, 304–318 (1971) Zbl 0222.31007 MR 0333220

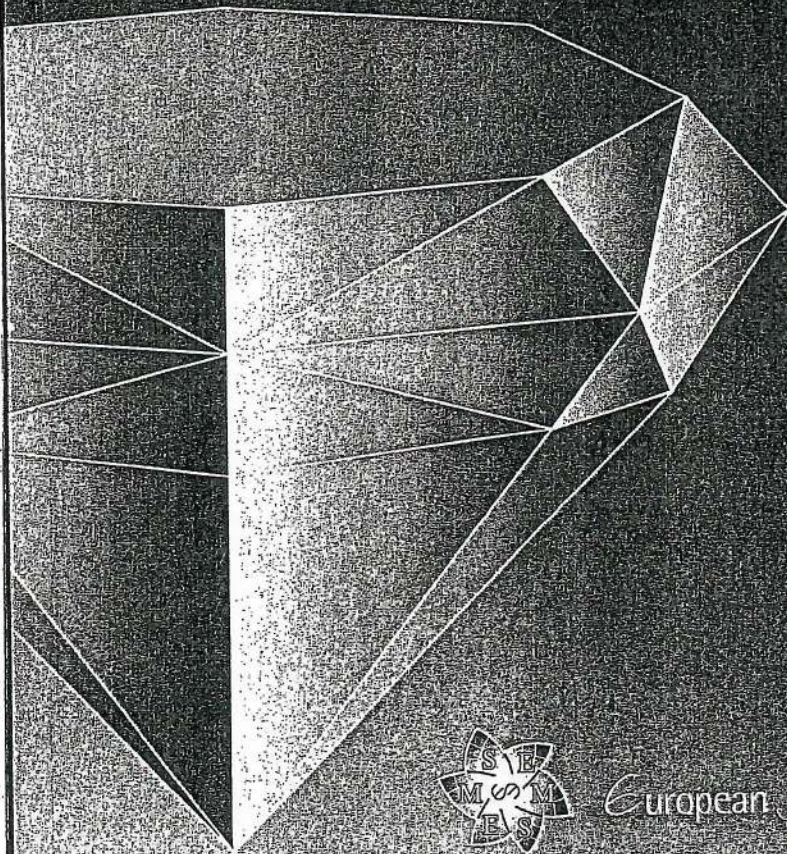
Sirakov, B.: Symmetry for exterior elliptic problems and two conjectures in potential theory. *Ann. Inst. H. Poincaré Anal. Non Linéaire* **18**, 135–156 (2001) Zbl 0997.35014 MR 1808026

Świąch, A.: $W^{1,p}$ -interior estimates for solutions of fully nonlinear, uniformly elliptic equations. *Adv. Differential Equations* **2**, 1005–1027 (1997) Zbl 1023.35509 MR 1606359

Wang, L.: On the regularity theory of fully nonlinear parabolic equations: I, II. *Comm. Pure Appl. Math.* **45**, 27–76 (1992) Zbl 0832.35025 MR 1135923

Vol. 9 / No. 2 / pp. 177–353 / 2007

Journal of the European Mathematical Society



European Mathematical Society

For information about Vols. 1-342, please contact your book-seller or Springer-Verlag.

Vol. 343: Algebraic K-Theory III, Hermitian K-Theory and Geometric Applications. Edited by H. Bass. XV, 572 pages. 1973.

Vol. 344: A. S. Troelstra (Editor), Metamathematical Investigation of Intuitionistic Arithmetic and Analysis. XVII, 485 pages. 1973.

Vol. 345: Proceedings of a Conference on Operator Theory. Edited by P. A. Fillmore. VI, 228 pages. 1973.

Vol. 346: Fučík et al., Spectral Analysis of Nonlinear Operators. II, 287 pages. 1973.

Vol. 347: J. M. Boardman and R. M. Vogt, Homotopy Invariant Algebraic Structures on Topological Spaces. X, 257 pages. 1973.

Vol. 348: A. M. Mathai and R. K. Saxena, Generalized Hypergeometric Functions with Applications in Statistics and Physical Sciences. VII, 314 pages. 1973.

Vol. 349: Modular Functions of One Variable II. Edited by W. Kuyk and P. Deligne. V, 598 pages. 1973.

Vol. 350: Modular Functions of One Variable III. Edited by W. Kuyk and J.-P. Serre. V, 350 pages. 1973.

Vol. 351: H. Tachikawa, Quasi-Frobenius Rings and Generalizations. XI, 172 pages. 1973.

Vol. 352: J. D. Fay, Theta Functions on Riemann Surfaces. V, 137 pages. 1973.

Vol. 353: Proceedings of the Conference on Orders, Group Rings and Related Topics. Organized by J. S. Hsia, M. L. Madan and T. G. Ralley. X, 224 pages. 1973.

Vol. 354: K. J. Devlin, Aspects of Constructibility. XII, 240 pages. 1973.

Vol. 355: M. Sion, A Theory of Semigroup Valued Measures. V, 140 pages. 1973.

Vol. 356: W. L. J. van der Kallen, Infinitesimally Central Extensions of Chevalley Groups. VII, 147 pages. 1973.

Vol. 357: W. Borho, P. Gabriel und R. Rentschler, Primideale in Einhüllenden auflösbarer Lie-Algebren. V, 182 Seiten. 1973.

Vol. 358: F. L. Williams, Tensor Products of Principal Series Representations. VI, 132 pages. 1973.

Vol. 359: U. Stammbach, Homology in Group Theory. VIII, 183 pages. 1973.

Vol. 360: W. J. Padgett and R. L. Taylor, Laws of Large Numbers for Normed Linear Spaces and Certain Fréchet Spaces. VI, 111 pages. 1973.

Vol. 361: J. W. Schutz, Foundations of Special Relativity: Kinematic Axioms for Minkowski Space-Time. XX, 314 pages. 1973.

Vol. 362: Proceedings of the Conference on Numerical Solution of Ordinary Differential Equations. Edited by D.G. Bettis. VIII, 490 pages. 1974.

Vol. 363: Conference on the Numerical Solution of Differential Equations. Edited by G. A. Watson. IX, 221 pages. 1974.

Vol. 364: Proceedings on Infinite Dimensional Holomorphy. Edited by T. L. Hayden and T. J. Suffridge. VII, 212 pages. 1974.

Vol. 365: R. P. Gilbert, Constructive Methods for Elliptic Equations. VII, 397 pages. 1974.

Vol. 366: R. Steinberg, Conjugacy Classes in Algebraic Groups (Notes by V. V. Deodhar). VI, 159 pages. 1974.

Vol. 367: K. Langmann und W. Lütkebohmert, Cousinverteilungen und Fortsetzungssätze. VI, 151 Seiten. 1974.

Vol. 368: R. J. Milgram, Unstable Homotopy from the Stable Point of View. V, 109 pages. 1974.

Vol. 369: Victoria Symposium on Nonstandard Analysis. Edited by A. Hurd and P. Loeb. XVIII, 339 pages. 1974.

Vol. 370: B. Mazur and W. Messing, Universal Extensions and One Dimensional Crystalline Cohomology. VII, 134 pages. 1974.

Vol. 371: V. Poenaru, Analyse Différentielle. V, 228 pages. 1974.

Vol. 372: Proceedings of the Second International Conference on the Theory of Groups 1973. Edited by M. F. Newman. VII, 740 pages. 1974.

Vol. 373: A. E. R. Woodcock and T. Poston, A Geometrical Study of the Elementary Catastrophes. V, 257 pages. 1974.

Vol. 374: S. Yamamuro, Differential Calculus in Topological Linear Spaces. IV, 179 pages. 1974.

Vol. 375: Topology Conference. Edited by R. F. Dickman Jr. and P. Fletcher. X, 283 pages. 1974.

Vol. 376: I. J. Good and D. B. Osteyee, Information, Weight of Evidence. The Singularity between Probability Measures and Signal Detection. XI, 156 pages. 1974.

Vol. 377: A. M. Fink, Almost Periodic Differential Equations. VIII, 336 pages. 1974.

Vol. 378: TOPO 72 - General Topology and its Applications. Proceedings 1972. Edited by R. A. Aló, R. W. Heath and J. Nagata. XIV, 651 pages. 1974.

Vol. 379: A. Badrikian et S. Chevet, Mesures Cylindriques, Espaces de Wiener et Fonctions Aléatoires Gaussiennes. X, 383 pages. 1974.

Vol. 380: M. Petrich, Rings and Semigroups. VIII, 182 pages. 1974.

Vol. 381: Séminaire de Probabilités VIII. Edité par P. A. Meyer. IX, 354 pages. 1974.

Vol. 382: J. H. van Lint, Combinatorial Theory Seminar Eindhoven University of Technology. VI, 131 pages. 1974.

Vol. 383: Séminaire Bourbaki - vol. 1972/73. Exposés 418-435. IV, 334 pages. 1974.

Vol. 384: Functional Analysis and Applications, Proceedings 1972. Edited by L. Nachbin. V, 270 pages. 1974.

Vol. 385: J. Douglas Jr. and T. Dupont, Collocation Methods for Parabolic Equations in a Single Space Variable (Based on C^1 -Piecewise-Polynomial Spaces). V, 147 pages. 1974.

Vol. 386: J. Tits, Buildings of Spherical Type and Finite BN-Pairs. X, 299 pages. 1974.

Vol. 387: C. P. Bruter, Eléments de la Théorie des Matroïdes. V, 138 pages. 1974.

Vol. 388: R. L. Lipsman, Group Representations. X, 166 pages. 1974.

Vol. 389: M.-A. Knus et M. Ojanguren, Théorie de la Descente et Algèbres d'Azumaya. IV, 163 pages. 1974.

Vol. 390: P. A. Meyer, P. Priouret et F. Spitzer, Ecole d'Eté de Probabilités de Saint-Flour III - 1973. Edité par A. Badrikian et P.-L. Hennequin. VIII, 189 pages. 1974.

Vol. 391: J. W. Gray, Formal Category Theory: Adjointness for 2-Categories. XII, 282 pages. 1974.

Vol. 392: Géométrie Différentielle, Colloque, Santiago de Compostela, Espagne 1972. Edité par E. Vidal. VI, 225 pages. 1974.

Vol. 393: G. Wassermann, Stability of Unfoldings. IX, 164 pages. 1974.

Vol. 394: W. M. Patterson, 3rd, Iterative Methods for the Solution of a Linear Operator Equation in Hilbert Space - A Survey. III, 183 pages. 1974.

Vol. 395: Numerische Behandlung nichtlinearer Integrodifferential- und Differentialgleichungen. Tagung 1973. Herausgegeben von R. Ansorge und W. Törnig. VII, 313 Seiten. 1974.

Vol. 396: K. H. Hofmann, M. Mislove and A. Stralka, The Pontryagin Duality of Compact O-Dimensional Semilattices and its Applications. XVI, 122 pages. 1974.

Vol. 397: T. Yamada, The Schur Subgroup of the Brauer Group. V, 159 pages. 1974.

Vol. 398: Théories de l'Information. Actes des Rencontres de Marseille-Luminy, 1973. Edité par J. Kampé de Fériet et C.-F. Picard. XII, 201 pages. 1974.

Lecture Notes in Mathematics

15

Edited by A. Dold and B. Eckmann

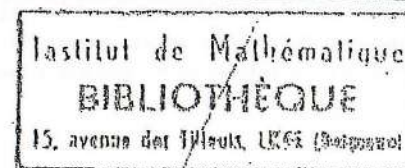
560

Combinatorial Mathematics IV

Proceedings of the Fourth Australian Conference
Held at the University of Adelaide
August 27-29, 1975

Université de Liège
BST - Sciences Appliquées et Mathématiques
1, Chemin des Chevreuils; Bât B52/4
B-4000 LIEGE

Edited by
Louis R. A. Casse and Walter D. Wallis



249P

Springer-Verlag
Berlin · Heidelberg · New York 1976

Editors

Louis R. A. Casse
Department of Pure Mathematics
University of Adelaide
South Australia 5000

Walter D. Wallis
Faculty of Mathematics
University of Newcastle
New South Wales/Australia 2308

Library of Congress Cataloging in Publication Data

Australian Conference on Combinatorial Mathematics, 4th,
University of Adelaide, 1975.
Combinatorial mathematics IV.

(Lecture notes in mathematics ; 560)
I. Combinatorial analysis--Congresses. I. Casse,
Louis R. A., 1934- II. Wallis, W. D. III. Title.
IV. Series: Lecture notes in mathematics (Berlin) ;
560.
QA3.L28 no. 560 [QA164] 510'.8s [511'.6] 76-54700

AMS Subject Classifications (1970): 05A10, 05A15, 05B05, 05B15,
05B20, 05B25, 05B30, 05C05, 05C20, 05C25, 05C30, 20B05,
62K10, 92A10, 94A10

ISBN 3-540-08053-8 Springer-Verlag Berlin · Heidelberg · New York
ISBN 0-387-08053-8 Springer-Verlag New York · Heidelberg · Berlin

This work is subject to copyright. All rights are reserved, whether the whole
or part of the material is concerned, specifically those of translation, re-
printing, re-use of illustrations, broadcasting, reproduction by photocopying
machine or similar means, and storage in data banks.

Under § 54 of the German Copyright Law where copies are made for other
than private use, a fee is payable to the publisher, the amount of the fee to be
determined by agreement with the publisher.

© by Springer-Verlag Berlin · Heidelberg 1976
Printed in Germany

PREFACE

The fourth Australian conference on combinatorial mathe-
matics was held at the University of Adelaide from 27th to 29th
August, 1975. The names of the fifty-two participants are listed
overleaf.

The conference was fortunate enough to hear addresses
given by distinguished combinatorialists from three countries:
L.J. Cummings (Canada), D.A. Preece (England) and J.S. Wallis
(Australia). This volume contains the texts of two of these
invited addresses and of twenty-three contributed talks. Manu-
scripts of the five remaining contributed talks given at the
conference are to be published elsewhere.

Many people helped with the organisation of the con-
ference and with the publication of this volume and we are
grateful to all of them. We particularly thank all those who
chaired sessions and refereed papers. We thank the University
of Adelaide for providing facilities, and in particular the
staff of the University Union. Particular thanks are extended
to Professor E.S. Barnes, the Deputy Vice-Chancellor.

Finally we thank Miss Anne Nicholls for typing this
manuscript.

L.R.A. Casse
W.D. Wallis

chsel, The Kronecker product of graphs, *Proc. Amer. Math. Soc.* 13
3), 47-52.

lson, On the adjacency matrix of a graph, in *Combinatorics* (Proc. Conf.
. Maths., Oxford) (Inst. Math. App. 1972), 295-321.

F Mathematics,

F Melbourne,

VIC.

16

p. 73 -- 92.

SOME COMPUTATIONAL RESULTS ON THE SPECTRA OF GRAPHS

C. GODSIL AND B. MCKAY

The polynomial of a graph is the characteristic polynomial of its 0-1 adjacency matrix. Two graphs are cospectral if their polynomials are the same.

In this paper some of the results from a numerical study of the polynomials of graphs are presented. The study has encompassed 9 point graphs, 9 point bipartite graphs, 14 point trees and 13 point forests. Also given are several theoretical results which were prompted by the numerical data. These include two characterizations of those cospectral graphs which have cospectral complements, and a proof that, in the sense of Schwenk [20] "almost no" trees are characterized by their polynomials together with the polynomials of their complements. In addition, mention is made of those cospectral graphs which have cospectral linegraphs, and those which are cospectral to their own complements.

1. INTRODUCTION

Graphs referred to in this paper have a finite, non-zero number of vertices and no loops or multiple edges. For such a graph G , \bar{G} refers to the complement of G , and $L(G)$ to the linegraph of G . For brevity, a graph on n vertices will be called an n -graph.

Suppose G is an n -graph. The *adjacency matrix* of G , also denoted G , is the $n \times n$ matrix whose (i,j) th entry is the number of edges from vertex i to vertex j . The *polynomial* of G , denoted $G(\lambda)$, is the characteristic polynomial of the adjacency matrix of G . An *eigenvalue* of G is a root of $G(\lambda)$. The eigenvalues of G , together with their multiplicities, constitute the *spectrum* of G . Two graphs which have the same polynomial, and hence the same spectrum are called *cospectral*.

Other graph theoretic concepts not defined here can be found in Harary [9] or in Behzad and Chartrand [2]. For any square matrix A , the trace of A is denoted $\text{tr } A$. J will always refer to a square matrix with each element one and I to an identity matrix.

The main purpose of this paper is to give the preliminary results of a computational study of the spectra of graphs. Previous studies of this kind have been made by Collatz and Singowitz [4] (5 point graphs and 8 point trees), King [13] (7 point graphs) and Mowshowitz [16] (10 point trees). In this study, the polynomials of 9 point graphs, 9 point bipartite graphs, 14 point trees and 13 point forests have been

Lecture Notes in Mathematics

For information about Vols. 1-342, please contact your bookseller or Springer-Verlag.

- Vol. 343: Algebraic K-Theory III, Hermitian K-Theory and Geometric Applications. Edited by H. Bass. XV, 572 pages. 1973.
- Vol. 344: A. S. Troelstra (Editor), Metamathematical Investigation of Intuitionistic Arithmetic and Analysis. XVII, 485 pages. 1973.
- Vol. 345: Proceedings of a Conference on Operator Theory. Edited by P. A. Fillmore. VI, 228 pages. 1973.
- Vol. 346: Fučík et al., Spectral Analysis of Nonlinear Operators. II, 287 pages. 1973.
- Vol. 347: J. M. Boardman and R. M. Vogt, Homotopy Invariant Algebraic Structures on Topological Spaces. X, 257 pages. 1973.
- Vol. 348: A. M. Mathai and R. K. Saxena, Generalized Hypergeometric Functions with Applications in Statistics and Physical Sciences. VII, 314 pages. 1973.
- Vol. 349: Modular Functions of One Variable II. Edited by W. Kuyk and P. Deligne. V, 598 pages. 1973.
- Vol. 350: Modular Functions of One Variable III. Edited by W. Kuyk and J.-P. Serre. V, 350 pages. 1973.
- Vol. 351: H. Tachikawa, Quasi-Frobenius Rings and Generalizations. XI, 172 pages. 1973.
- Vol. 352: J. D. Fay, Theta Functions on Riemann Surfaces. V, 137 pages. 1973.
- Vol. 353: Proceedings of the Conference on Orders, Group Rings and Related Topics. Organized by J. S. Hsia, M. L. Madan and T. G. Ralley. X, 224 pages. 1973.
- Vol. 354: K. J. Devlin, Aspects of Constructibility. XII, 240 pages. 1973.
- Vol. 355: M. Sion, A Theory of Semigroup Valued Measures. V, 140 pages. 1973.
- Vol. 356: W. L. J. van der Kallen, Infinitesimally Central Extensions of Chevalley Groups. VII, 147 pages. 1973.
- Vol. 357: W. Borho, P. Gabriel und R. Rentschler, Primideale in Einhüllenden auflösbarer Lie-Algebren. V, 182 Seiten. 1973.
- Vol. 358: F. L. Williams, Tensor Products of Principal Series Representations. VI, 132 pages. 1973.
- Vol. 359: U. Stambach, Homology in Group Theory. VIII, 183 pages. 1973.
- Vol. 360: W. J. Padgett and R. L. Taylor, Laws of Large Numbers for Normed Linear Spaces and Certain Fréchet Spaces. VI, 111 pages. 1973.
- Vol. 361: J. W. Schutz, Foundations of Special Relativity: Kinematic Axioms for Minkowski Space-Time. XX, 314 pages. 1973.
- Vol. 362: Proceedings of the Conference on Numerical Solution of Ordinary Differential Equations. Edited by D.G. Bettis. VIII, 490 pages. 1974.
- Vol. 363: Conference on the Numerical Solution of Differential Equations. Edited by G. A. Watson. IX, 221 pages. 1974.
- Vol. 364: Proceedings on Infinite Dimensional Holomorphy. Edited by T. L. Hayden and T. J. Suffridge. VII, 212 pages. 1974.
- Vol. 365: R. P. Gilbert, Constructive Methods for Elliptic Equations. VII, 397 pages. 1974.
- Vol. 366: R. Steinberg, Conjugacy Classes in Algebraic Groups (Notes by V. V. Deodhar). VI, 159 pages. 1974.
- Vol. 367: K. Langmann and W. Lütkebohmert, Cousinverteilungen und Fortsetzungssätze. VI, 151 Seiten. 1974.
- Vol. 368: R. J. Milgram, Unstable Homotopy from the Stable Point of View. V, 109 pages. 1974.
- Vol. 369: Victoria Symposium on Nonstandard Analysis. Edited by A. Hurd and P. Loeb. XVIII, 339 pages. 1974.
- Vol. 370: B. Mazur and W. Messing, Universal Extensions and One Dimensional Crystalline Cohomology. VII, 134 pages. 1974.
- Vol. 371: V. Poenaru, Analyse Différentielle. V, 228 pages. 1974.
- Vol. 372: Proceedings of the Second International Conference on the Theory of Groups 1973. Edited by M. F. Newman. VII, 740 pages. 1974.
- Vol. 373: A. E. R. Woodcock and T. Poston, A Geometrical Study of the Elementary Catastrophes. V, 257 pages. 1974.
- Vol. 374: S. Yamamuro, Differential Calculus in Topological Linear Spaces. IV, 179 pages. 1974.
- Vol. 375: Topology Conference. Edited by R. F. Dickman Jr. and P. Fletcher. X, 283 pages. 1974.
- Vol. 376: I. J. Good and D. B. Osteyee, Information, Weight of Evidence. The Singularity between Probability Measures and Signal Detection. XI, 156 pages. 1974.
- Vol. 377: A. M. Fink, Almost Periodic Differential Equations. VIII, 336 pages. 1974.
- Vol. 378: TOPO 72 - General Topology and its Applications. Proceedings 1972. Edited by R. A. Alb, R. W. Heath and J. Nagata. XIV, 651 pages. 1974.
- Vol. 379: A. Badrikian et S. Chevel, Mesures Cylindriques, Espaces de Wiener et Fonctions Aléatoires Gaussiennes. X, 383 pages. 1974.
- Vol. 380: M. Petrich, Rings and Semigroups. VIII, 182 pages. 1974.
- Vol. 381: Séminaire de Probabilités VIII. Edité par P. A. Meyer. IX, 354 pages. 1974.
- Vol. 382: J. H. van Lint, Combinatorial Theory Seminar Eindhoven University of Technology. VI, 131 pages. 1974.
- Vol. 383: Séminaire Bourbaki - vol. 1972/73. Exposés 418-435. IV, 334 pages. 1974.
- Vol. 384: Functional Analysis and Applications, Proceedings 1972. Edited by L. Nachbin. V, 270 pages. 1974.
- Vol. 385: J. Douglas Jr. and T. Dupont, Collocation Methods for Parabolic Equations in a Single Space Variable (Based on C^1 Piecewise-Polynomial Spaces). V, 147 pages. 1974.
- Vol. 386: J. Tits, Buildings of Spherical Type and Finite BN-Pairs. X, 299 pages. 1974.
- Vol. 387: C. P. Bruter, Eléments de la Théorie des Matroides. V, 138 pages. 1974.
- Vol. 388: R. L. Lipsman, Group Representations. X, 166 pages. 1974.
- Vol. 389: M.-A. Knus et M. Ojanguren, Théorie de la Descente et Algèbres d'Azumaya. IV, 163 pages. 1974.
- Vol. 390: P. A. Meyer, P. Priouret et F. Spitzer, Ecole d'Été de Probabilités de Saint-Flour III - 1973. Edité par A. Badrikian et P.-L. Hennequin. VIII, 189 pages. 1974.
- Vol. 391: J. W. Gray, Formal Category Theory: Adjointness for 2-Categories. XII, 282 pages. 1974.
- Vol. 392: Géométrie Différentielle, Colloque, Santiago de Compostela, Espagne 1972. Edité par E. Vidal. VI, 225 pages. 1974.
- Vol. 393: G. Wassermann, Stability of Unfoldings. IX, 164 pages. 1974.
- Vol. 394: W. M. Patterson, 3rd, Iterative Methods for the Solution of a Linear Operator Equation in Hilbert Space - A Survey. III, 183 pages. 1974.
- Vol. 395: Numerische Behandlung nichtlinearer Integrodifferential- und Differentialgleichungen. Tagung 1973. Herausgegeben von R. Ansorge und W. Törnig. VII, 313 Seiten. 1974.
- Vol. 396: K. H. Hofmann, M. Mislove and A. Stralka, The Pontryagin Duality of Compact 0-Dimensional Semilattices and its Applications. XVI, 122 pages. 1974.
- Vol. 397: T. Yamada, The Schur Subgroup of the Brauer Group. V, 159 pages. 1974.
- Vol. 398: Théories de l'Information, Actes des Rencontres de Marseille-Luminy, 1973. Edité par J. Kampé de Fériet et C.-F. Picard. XII, 201 pages. 1974.

MAT: COL: LNM: 0560

Lecture Notes in Mathematics

Edited by A. Dold and B. Eckmann

560

Combinatorial Mathematics IV

Proceedings of the Fourth Australian Conference
Held at the University of Adelaide
August 27-29, 1975

Université de Liège
BST - Sciences Appliquées et Mathématiques
1, Chemin des Chevreuils; Bât B52/4
B-4000 LIEGE

Edited by
Louis R. A. Casse and Walter D. Wallis



Institut de Mathématique
BIBLIOTHÈQUE
15, avenue des Thuilleries, LIEGE (Belgique)

Springer-Verlag
Berlin Heidelberg New York 1976

UNIVERSITE DE PARIS-SUD
CENTRE D'ORSAY

THESE

présentée

Pour obtenir

Le TITRE..... de DOCTEUR 3^o CYCLE.....

SPECIALITE : STATISTIQUES

PAR

M. ASTIER Roger

SUJET : METHODE BOX JENKINS APPLIQUEE AUX SERIES DE TRANSPORT

soutenue le 26. FEVRIER 1982 devant la Commission d'examen

MM. DACUNHA CASTELLE..... Président

DUFLO.....

GUYON.....

.....

.....

176 p.

6/1982